Textbook Reference: Sections 24.1/24.2 and 24.8 (Interference in Double-Slits & Diffraction Gratings)

Goal/Objectives:
- Identify conditions necessary for interference patterns to be observed
- Draw and label diagrams showing double-slit interference patterns
- Analyze problems using double-slit and diffraction grating interference patterns
- Understand when the small angle approximation can be applied to interference patterns

(SmartBoard):
- particle/wave properties slide

- so, we're finally ready for interference!
- let's look again at the laser through the little slits . . . how does interference explain this pattern?

Demo:
- red laser through diffraction grating in 300 lines/mm

- however, it is obvious that the simplest way to learn the pattern is by looking at how only 2 rays interfere, so we will begin with this

(SmartBoard) Young's Double Slit Interference:

- to see how the interference occurs, we will use a diagram
- we always use the same diagram to show the parts of the interference . . . so know it well!!!
- also give yourself plenty of room . . . there's a lot to draw & label

- it is looking down on the experiment . . . so imagine if we were looking at our laser from the ceiling

- the laser is drawn on the left side of the diagram . . .
- in front of the laser is our slide w/ the holes (labeled as the "Slits")
- again, there is a large variety of how many slits that can be used . . . remember we were using 300 slits per mm earlier
- however, on our diagram, we will always draw 2 gaps to show 2 holes . . . mostly b/c we don't want to try to draw all 300 holes or 300 different rays of light!

- at the far end we draw something called the screen . . . this is any surface that we can use to see the interference pattern on
- so, our screen was actually just the wall of the classroom

- now that the diagram is up, try to picture it in relation to the actual experiment that we did . . .
- basically, we are taking the laser & turning it sideways . . . then looking down on the top of the pattern

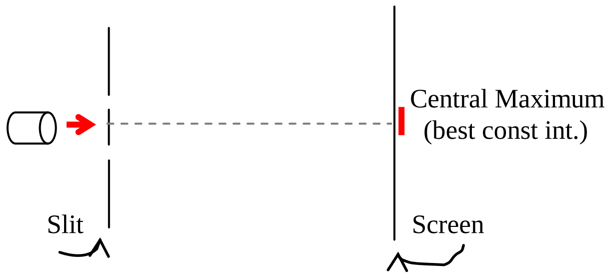
Homework: C #4, 8, 15
P #10, 8, 3, 4, 7, 39, 41, 43ab

Video: Double-Slit Interference

Equipment: Red laser pen
Green laser pen
Diffraction grating

Double slit interference web site
(Reference for general patterns)

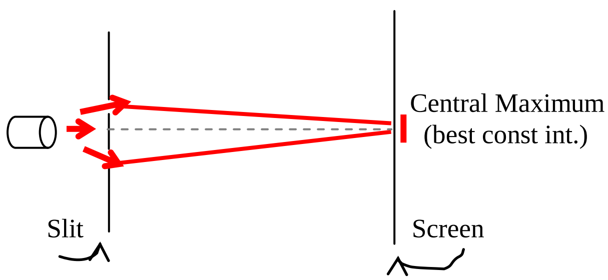
(SmartBoard) Young's Double Slit Interference:



- the next thing to add to our diagram is the constructive interference spots of light
- remember that there was one spot right in the center . . .
- this constructive interference spot is called the "central maximum" . . .
- it occurs in every interference pattern for light

- it is always centered across from the center of the laser . . . and it is always the best area of constructive interference
- this means that it will be the brightest of all the constructive interference areas
- so, picture an axis centered in the middle of the 2 slits and extending across to the screen . . . the central max will be located on this axis

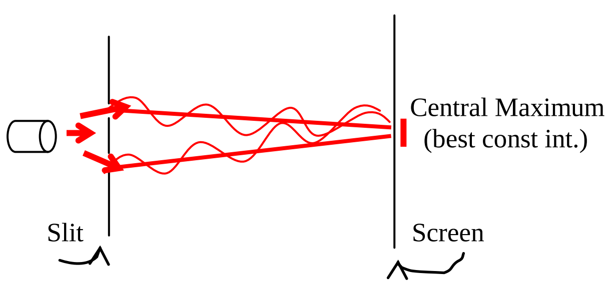
(SmartBoard) Young's Double Slit Interference:



- so, what causes the interference for the central max?
- remember that our slit split the laser up into 2 rays . . .
- now, our slit is not exactly drawn to scale . . . but both rays will have to angle slightly inwards to reach the central max
- this will allow both rays to strike the screen at exactly the center of the pattern

- now, also remember that these rays really have crests and troughs as they travel
- we need to add these to our diagram too . . . although this will really not be to scale ☺!
- suppose that both rays leave the laser beginning at a crest . . . and suppose the top ray goes through 6 crests before it reaches the screen
- the bottom ray is traveling exactly the same distance . . . so the bottom ray also must go through 6 crests before it reaches the screen

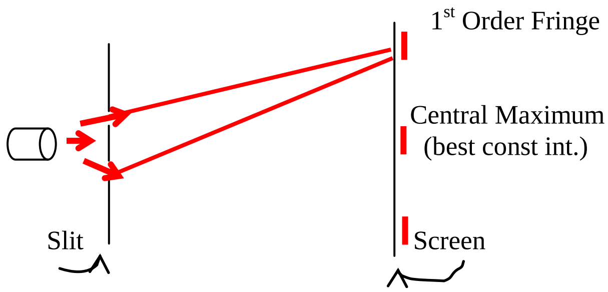
(SmartBoard) Young's Double Slit Interference:



- if both rays leave the laser at a crest, and both rays reach the screen at a crests . . . then we have constructive interference . . .
- now, what if the top ray was actually at a trough when it reaches the screen . . . the bottom ray would also be at a trough when it reaches the screen . . .
- what type of interference do we get when we have 2 troughs together . . . also constructive interference!

- so, b/c the 2 rays are always traveling equal distances to get to the central maximum . . . the 2 rays will always line up equally . . .
- and we have a guarantee that the 2 rays will always have constructive interference at the center
- this is true then for all interference patterns . . . regardless of the wavelength of light or the number of slits used to create the pattern

(SmartBoard) Young's Double Slit Interference:



- so, what about the other constructive interference points? . . . this gets a little more complicated!
- we will use the same diagram . . . although you might want to redraw it so that it doesn't get too crowded w/ light rays

- this time, we are looking at a constructive interference spot that is out away from the central maximum . . .
- this could be to the right or the left on our example w/ the laser . . . on our diagram, that would be represented by a spot above or a spot below
- this spot is called the 1^{st} order fringe . . . (something like the 1^{st} overtone!)
- it is the next place where constructive interference occurs

(B3 Col1) Variables:

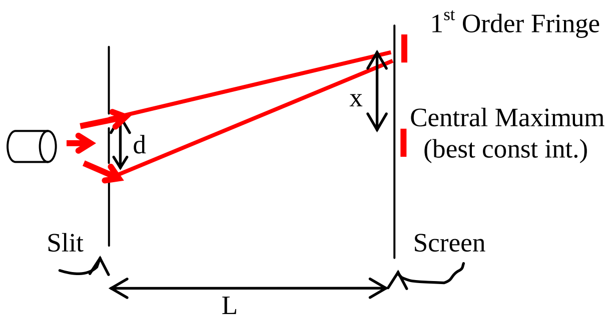
λ = wavelength
L = length from slits to screen
d = slit separation
x = distance from central max to fringe

- before we go any further, we need to identify some of the distances that will be important . . .
- b/c there are so many distances, we have lots of variables to keep track of . . .

- one obvious one is the wavelength of light . . .

- the first important distance on our <u>diagram</u> is the length from the laser to the screen . . . abbreviated L

(SmartBoard) Young's Double Slit Interference:



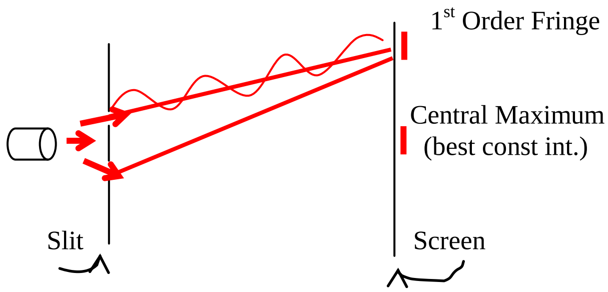
- the next important distance is the distance between the slits . . . this could be the distance between just 2 slits . . . or if we are using many slits like we did w/ our laser, it would be the distance between each adjacent pair of slits
- this is usually known as the slit separation

- the distance between the central maximum and the next bright spot of light is known as x (probably b/c they ran out of other variables to use for a length ☺)

- so, how do we figure out exactly what this distance *x* is . . . in other words, what determines where that next bright spot is going to be?
- it is determined by several factors . . .
- remember that the 2 rays will again need to meet at the screen to produce this next bright spot . . .

- however, do the 2 rays travel the same distance this time?
- no, both rays are angled up this time, instead of towards the middle
- so, the bottom ray has to travel a <u>greater distance</u> to get to the next position of bright light!
- that extra distance is known as the <u>path difference</u> . . . and it will be used to help us calculate the distance *x*

<u>(SmartBoard) Young's Double Slit Interference:</u>



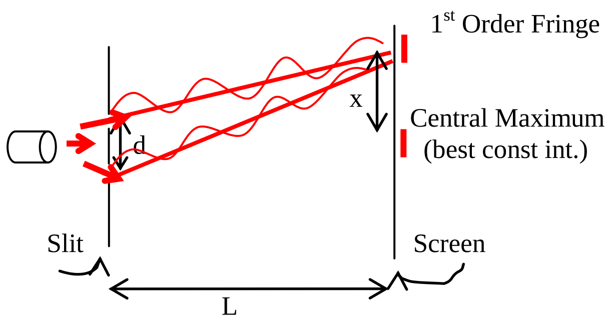
- so, suppose that we draw in the actual wavelength for the top ray (again . . . not to scale!)
- suppose that our top wave gets to the screen in 6 crests . . . so, it leaves the laser at a crest & reaches the screen in a crest
- if we draw the bottom wave leaving the laser at a crest . . . and we go for 6 crests, will the bottom ray be at the screen?
- no, 6 crests for the bottom ray will leave it short of the screen
- (obviously, the number would be much greater than 6 . . . remember we are talking about wavelengths of 600 nm here . . . but the same thing is happening, just on a very very small scale)

- so, if the bottom wave must travel a greater distance, is there any way that the bottom wave could still reach the screen at a crest?
- the bottom wave has to travel an extra wavelength!
- so, in order to make up for the fact that the bottom ray must travel a greater distance, it has to travel the exact amount so that it gets to the next crest!

- we can see this pattern occurring. . .
- first, we have the central max shown
- both rays of light travel through 6 crests (shown in dark red)
- they both travel the same distance to the center, so crests are still matched up with crests . . . and they constructively interfere

- however, if we look at the how the rays compare to each other as we move away from the central max, we see that the rays no longer travel the same distance . . . so the crests no longer match up between the rays
- finally, when we get far enough away . . . the crests from the top ray are finally able to match up again with crests from the bottom ray . . .
- but the top ray has gone through 6 crests while the bottom ray had to travel 7!

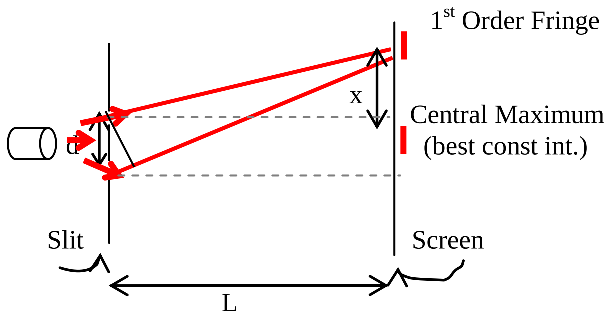
<u>(SmartBoard) Young's Double Slit Interference:</u>



- this leads us to our first important fact about the rays . . . the difference in the distance traveled (or the path difference) must be equal to a whole number of wavelengths in order for constructive interference to work . . .
- so, the bottom ray must travel exactly 1 wavelength, or maybe 2 wavelengths, or 3 . . . etc extra distance in order to get back to the point where its crests line up with crests from the top ray again

(SmartBoard) Young's Double Slit Interference:

- slides showing path difference & angles added



- by adding a little geometry to our diagram, we can use this fact to find our equation for interference . . .

- now remember that this diagram is not to scale . . . d is very very small . . . so it's almost like the <u>2 rays are parallel</u> to each other (might skip to the last page of SB file to show this)
- so, we are going to add an axis from each individual slit to the screen
- then, we measure the angle from each ray to the axis . . . and the angles are the same (again, remember that our diagram is really not to scale . . . we have drawn our rays with a very large separation so we can see the difference between them) . . . but if these 2 rays are almost parallel to each other, then the angles should be equal, even if they don't look like it here

- then, we draw in a line from the beginning of the top ray & striking the bottom ray perpendicularly
- this forms a little tiny right triangle next to the slit

- if the rays are almost parallel, then the bottom of the little triangle, (the green side) represents the path difference
- and from geometry, the angle on the top of that little triangle should be congruent to the other 2 angles we have measured

- from trig, the bottom side of this little triangle . . . the side that represents the path difference . . . can be found then by taking d (hypotenuse of little triangle) multiplied by sin θ
- and this leads us (finally!) to an equation that relates the quantities for this interference pattern!

(B3 Col2) Equations:

- from path difference: $\underbrace{d \cdot \sin\theta} = \underbrace{m \cdot \lambda}$

path difference

$m = 1, 2 \ldots$ for const

$m = \frac{1}{2}, 1\frac{1}{2}, 2\frac{1}{2} \ldots$ for dest

- the path difference . . . or the extra distance that 1 ray must travel . . . can be found in 2 ways . . .
- by d sin θ (from our little triangle)
- or, by an integer times the wavelength (because the bottom ray must travel a whole number of extra wavelengths in order for its crests to line up with the crests of the top ray again . . .

- so, our first equation is $d \cdot \sin\theta = m \cdot \lambda$
- notice something slightly confusing about this equation . . .
- there is no actual variable for "path difference"
- so, often a question might ask you to calculate the path difference of an interference pattern
- you must remember that the produce on either side of the equation will give you path difference!

(B3 Col2) Equations:

- from path difference: $d \cdot \sin\theta = m \cdot \lambda$

path difference

m = 1, 2 . . . for const

m = ½ , 1 ½, 2 ½ . . . for dest

- also note that the wavelength is simply multiplied by the variable "m"
- m represents integer values . . . 1, 2, 3, etc for constructive interference
- b/c any situation where there is an integer number of extra wavelengths would allow the 2 waves to be back in phase with each other

- but, replacing m with ½ an extra wave . . . for instance adding ½ a wave, or 1 ½ extra waves, etc . . . would cause the crests of the top ray to line up with the troughs of the bottom ray . . .
- thus, m = ½ would show us positions where destructive interference occurs

(SmartBoard) Young's Double Slit Interference:

- slides showing 2 rays originating from approximately the same point

- now this equation is relatively simple to use (and on your green sheet) . . . but it has one major problem
- it does not include x!!!

(B3 Col2) Equations:

- from path difference: $d \cdot \sin\theta = m \cdot \lambda$

path difference

m = 1, 2 . . . for const

m = ½ , 1 ½, 2 ½ . . . for dest

- from geometry: $\tan\theta = \dfrac{x}{L}$

- it gives us the angle at which the light bends to create the interference pattern
- if we change our diagram slightly (last pg on SB file), we can see how this equation will help us
- remember that d is so small that it is almost as if the 2 rays are coming from the same location
- so, we can measure this angle as being either θ_1 or θ_2

- and, if we look at the larger triangle that is formed, we see another geometric relationship . . .
- the <u>tangent</u> of this angle is x / L

- now, here we need to use a fact of math that you might now have ever considered before . . .
- take your calculators (be sure they are in degrees) . . . and choose an angle less than 20
- type in the tangent of that angle & see what it is . . . then type in the sine of that angle & see what it is . . .
- they should be almost identical!

(SmartBoard) Small Angle Approximation:

$$d \cdot \sin\theta = m \cdot \lambda$$

$$d \cdot \frac{x}{L} = m \cdot \lambda$$

- for relatively small angles . . . (often approximated as less than 20˚) . . . the sine and the tangent can be considered the same thing!
- so, we can replace the sin θ in our original equation with x/L (which is equal to tan θ)
- this equation now includes x (which is very very useful!)

(B3 Col2) Equations:

- from path difference: $\underbrace{d \cdot \sin\theta}_{\text{path difference}} = m \cdot \lambda$

path difference

m = 1, 2 . . . for const

m = ½ , 1 ½, 2 ½ . . . for dest

- from geometry: $\tan\theta = \frac{x}{L}$

- for small θ's: $x_m \approx \frac{m \cdot \lambda \cdot L}{d}$

- it is usually rearranged to be in the form x = . . .
- it is also included on your green sheet . . . but note the "approximately equal to" sign!

- this version of the equation should only be used for relatively small angles . . . less than 20˚
- if the angle is greater than 20˚ . . . then this version of the equation may give results that are not quite as accurate as they should be . . .

- if the angle is significantly greater than 20˚ . . . you should avoid this version of the equation altogether
- you can still calculate the things you need . . . but you will have to use a combination of the top 2 equations instead . . .
- i.e. use the top equation to find the angle . . . then use the tangent equation to find x or vice versa . . .

Demo:

- red laser through diffraction grating in 300 lines/mm
- then red laser through 600 and 100 lines/mm

- green laser through 300 lines/mm

- now, this equation can be used to identify some general trends . . .
- for instance, what happens if d becomes larger or d becomes smaller . . .
- what happens if L is longer or shorter? . . .
- and what happens if the wavelength is longer or shorter?

- more detail coming tomorrow on how these 2 equations work together!

References:

http://www.walter-fendt.de/ph14e/doubleslit.htm

Announcements: